AN ANALYTIC MODEL THAT SHOWS POSSIBLE BIAS IN OPACITY MEASUREMENTS

Todd Urbatsch

Los Alamos National Laboratory
P.O. Box 1663, Los Alamos, New Mexico, 87545
tmonster@lanl.gov

In the High Energy Density regime where thermal X-rays are dominant (thermal radiation transport and hydrodynamics), experiments are conducted to validate the absorption opacities. For material temperatures in the range of 100-200eV, the Opacity-on-Z experiments have been producing observed opacities that, in some cases, are twice as large as what theory predicts. We present a simple analytic model that shows that, if the drive source is not laterally uniform and wobbles from shot-to-shot, the inferred opacities may be larger than the actual opacities.

I. INTRODUCTION

For over a decade, the Opacity-on-Z experiments reproducibly have given unpredictable comparisons to theoretical predictions of opacity values. For iron at a temperature in the range of 170-190eV, the experimentally inferred opacity is nearly twice as large as theory predicts.[1] For temperatures of about 156eV, the experiment matches theory. The platform at the Z-pinch facility at Sandia National Laboratories drives a large current through two, nested cylindrical array of tungsten wires that collapse on and compress a cylindrical foam core. The foam core heats up and emits radiation that, in early time, heats and expands the plastic-covered foil, and then serves as the continuing source of backlighting radiation. See Figure 1. The experimental analysis compares thermal radiation through a sub-micron-thick, half-moon foil of iron to that through a no-foil control opposite the half-moon. Actually, the foil is typically an FeMg alloy because magnesium has well-known lines to diagnose the temperature and density of the foil. Simply put, the opacity of the iron, or other target material, is inferred from the log of the ratio of the through-foil intensity spectrum to the control intensity spectrum. The inference relies on the purely absorbing, mono-directional transport equation, [2]

$$I = I_0 e^{-\sigma x} = I_0 e^{-\rho\kappa x}\,, \qquad (1)$$

where I is the intensity emerging from the foil, I_0 the intensity impinging on the foil, x [cm] is the foil thickness, and where the absorption opacity, σ [cm^{-1}], is also written as the material density, ρ [g/cm^3], times the absorption coefficient, κ [cm^2/g].

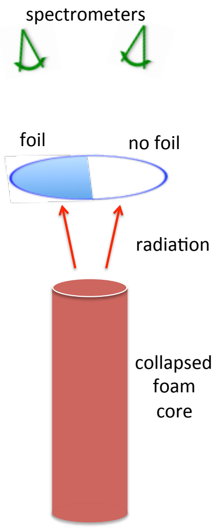

Figure 1. Notional schematic of the Opacity-on-Z experiments. The foil is about 4.5mm in diameter, and the spectrometers are about 4m away.

The analysis to infer experimental opacities is complicated and time-consuming, and it involves removing backgrounds and averaging over several experimental shots.[1] Many hypotheses have been proposed for the iron opacity discrepancies, including missing physics from the theory, but it remains a mystery. A hypothesis related to the experimental platform is that the foil shields the material behind it, which remains cooler and optically thicker than on the no-foil side, and the overall increased opacity gets attributed to the foil.[3] Several potential issues with the experimental platform have been investigated with 1D radiation-hydrodynamic calculations, with all issues being discounted.[4]

II. HYPOTHESIS

Looking at the backlighting-phase pin-hole camera images in Bailey's article[5] and the angular intensity in Lemke's article,[6] it appears that the radiation source emitted from the Z-pinch foam core is not uniform side-to-side and could wobble from shot-to-shot. Our hypothesis is that taking averages of nonlinear functions of non-uniform sources could give a bias in the inferred opacity.

III. MODEL AND RESULTS

The analytic model, shown in Figure 2, is two side-by-side simple analytic transmissions, one through a foil of thickness x and one through a void. If $I_0 = I_{00}$, the source is uniform foil-side-to-void-side, and this bias goes away.

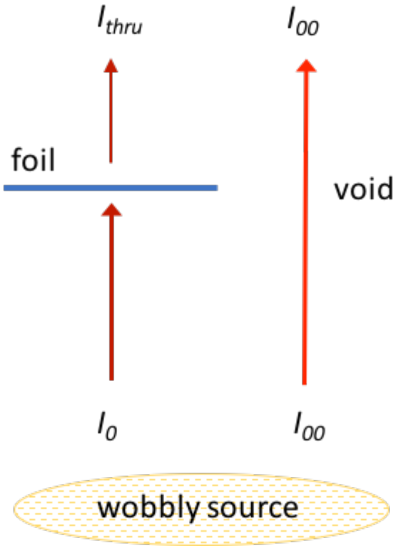

Figure 2. Schematic of an analytic model with transmission through a foil of thickness x and through a void when the sources to each are not the same.

Let us assume that the source wobble is 30%,[6] and that I_0 varies as a function of I_{00},

$$I_0 = \alpha I_{00}\ , \tag{2}$$

where $0.7 < \alpha < 1.3$ and α is uniformly distributed. In the experiment, we will only observe I_{thru} and I_{00}, and we will have no direct knowledge of I_0.

III.A. Constant Foil Opacity

First, we assume that the foil opacity, σ, has no dependence on the temperature of the foil. This case represents the bound-free continuum in the iron opacity. In our model, referring to I_{thru} as I without the subscript, the observed opacity will be inferred from

$$I_{thru} = I = I_{00}\,\mathrm{e}^{-\sigma_{obs}x}\ , \tag{3}$$

such that

$$\begin{aligned}\sigma_{obs} &= -\frac{1}{x}\log\frac{I}{I_{00}}\\ &= -\frac{1}{x}\left(\log I - \log I_{00}\right)\end{aligned} \tag{4}$$

but the actual opacity is

$$\begin{aligned}\sigma &= -\frac{1}{x}\log\frac{I}{I_0} = -\frac{1}{x}\log\frac{I}{\alpha I_{00}}\\ &= -\frac{1}{x}\left(\log I - \log I_{00} - \log\alpha\right)\\ &= \sigma_{obs} + \frac{1}{x}\log\alpha\end{aligned} \tag{5}$$

We can look at the difference,

$$\sigma_{obs} - \sigma = -\frac{1}{x}\log\alpha\,. \tag{6}$$

Since x is always positive, and since we have assumed an average over multiple experiments and that α is uniformly distributed between 0.7 and 1.3, then, given that log(0.7) = -0.357 and log(1.3) = 0.262, the observed opacity will be higher than the actual by the range of

$$\sigma_{obs} - \sigma = \frac{1}{x}\left[-0.262, 0.357\right]. \tag{7}$$

From the midpoint of the extremes, our observed opacity may be approximately 0.05/x too high. Analytically averaging the coefficient

$$-\left\langle\log(\alpha)\right\rangle = -\int_{1-\alpha}^{1+\alpha}\log(\alpha')d\alpha' / \int_{1-\alpha}^{1+\alpha} d\alpha' \tag{8}$$

via the definite integral of the natural log, $y(\log y) - y$ for $0.7<y<1.3$, and dividing by (1.3-0.7), the average difference is about 0.015/x too high. A numerical integration/averaging matches the analytic result. **Figure 3** shows the value of the average of the natural logarithm for varying values of source wobble.

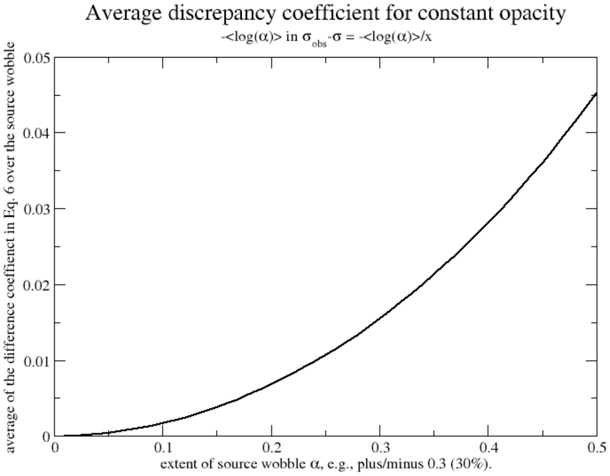

Figure 3. Bias in taking the average of the coefficient in the opacity difference formula for constant opacity.

III.B. Temperature-Dependent Foil Opacity

Now, let us assume that the opacity of the foil is inversely proportional to the temperature of the foil,

$$\sigma = \frac{K'}{T^3}, \tag{9}$$

Where *K'* is the coefficient we are trying to determine from our experiment, and *T* is the temperature of the foil. Presumably, we will know the temperature from an analysis of the magnesium lines. Let us further assume a Planckian spectral distribution for the intensity so that we can equate the temperature and intensity,

$$I = aT^4, \tag{10}$$

so that

$$T = \left(\frac{I}{a}\right)^{1/4}. \tag{11}$$

Then

$$\sigma = \frac{K'}{T^3} = \frac{K' a^{3/4}}{I^{3/4}} = \frac{K}{I^{3/4}}, \tag{12}$$

where we have absorbed the power of the radiation constant, *a*, into the coefficient, *K*.

Let us calculate the ratio K_{obs}/K. For the observed coefficient K_{obs}, we have

$$\begin{aligned} I_{thru} &= I_{00}\, \mathrm{e}^{-\sigma_{obs} x} \\ &= I_{00} e^{-xK_{obs}/I_{00}^{3/4}} \end{aligned}. \tag{13}$$

Then

$$\log\left(I_{thru}/I_{00}\right) = {-xK_{obs}}\Big/{I_{00}^{3/4}}, \tag{14}$$

and

$$K_{obs} = -\frac{1}{x} I_{00}^{3/4} \log\left(I_{thru}/I_{00}\right). \tag{15}$$

For the actual coefficient, *K*, we have

$$\begin{aligned} I_{thru} &= I_0\, \mathrm{e}^{-\sigma x} \\ &= I_0 e^{-xK/I_0^{3/4}} \\ &= \alpha I_{00} e^{-xK/(\alpha I_{00})^{3/4}} \end{aligned}. \tag{16}$$

Then

$$\log\left(I_{thru}/(\alpha I_{00})\right) = {-xK}\Big/{(\alpha I_{00})^{3/4}} \tag{17}$$

and

$$K = -\frac{1}{x}(\alpha I_{00})^{3/4} \log\left(I_{thru}/(\alpha I_{00})\right). \tag{18}$$

The ratio K_{obs}/K is

$$\begin{aligned} \frac{K_{obs}}{K} &= \frac{\log\left(I_{thru}/I_{00}\right)}{\alpha^{3/4}\log\left(I_{thru}/(\alpha I_{00})\right)} \\ &= \frac{\log\left(I_{thru}/I_{00}\right)}{\alpha^{3/4}\left(\log\left(I_{thru}/I_{00}\right) - \log\alpha\right)} \end{aligned}. \tag{19}$$

In **Figure 4**, we plot this ratio of the observed opacity coefficient to the actual opacity coefficient as a function of the ratio of the observed intensity through the foil to that through the void. We plot the ratio for the extremes of α, 0.7 and 1.3. If α=1, there is no bias. The way to read this plot is to find out what your "experiment" said the transmission fraction was. If, for example, the foil absorbed 40% of the intensity, then $I_{\mathrm{thru}}/I_{00} = 0.6$ on the x-axis of the plot. If, for that particular experiment, the source were high by 30% on the foil side, the observed opacity coefficient would be low by almost half, as seen in the red dashed line. If the source were low by 30% on the foil side, the observed opacity coefficient would be high by more than a factor of four. **Figure 4** shows that the two curves representing the α limits are not centered about unity. Numerically integrating and averaging the distribution for each value of I_{thru}/I_{00} is shown as the magenta line in **Figure 4**. Thus, averaging experiments would tend to overestimate the opacity.

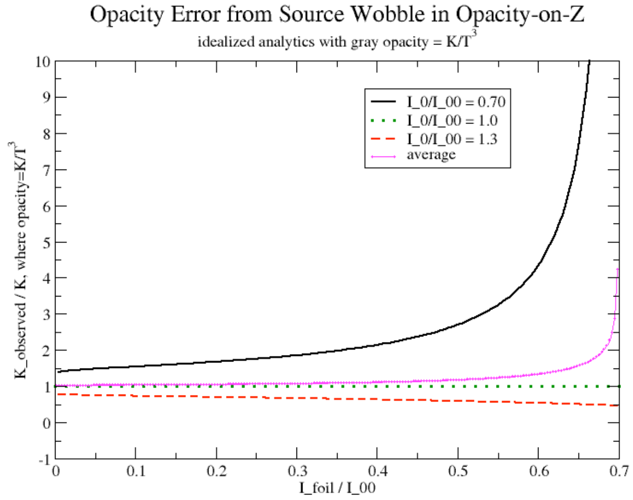

Figure 4. The bias in the observed opacity coefficient when the drive is not uniform side-to-side.

The curves lose meaning as I_{thru}/I_{00} approaches zero, which physically represents no transmission through the foil whatsoever. The black curve goes to infinity as I_{thru}/I_{00} approaches our lower source non-uniformity fraction of 0.7, a situation that physically represents the absence of a foil. These no-information and low-signal behaviors are related to why the real experiments target a foil transmission between 0.2 and 0.8.[4]

For a 30% wobble, and for three selected values of $\alpha = I_{\text{thru}}/I_{00} = 0.2$, 0.5, and 0.675, the distribution of bias is shown in **Figure 5**. For these three values of α, the biases in the average of coefficient ratios are 6%, 18%, and 87% high, respectively.

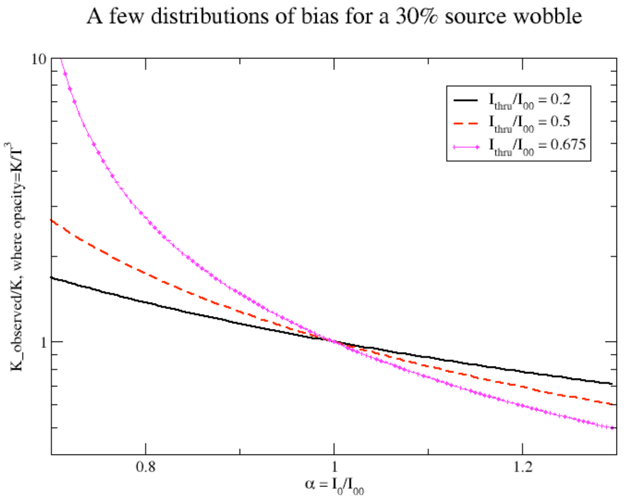

Figure 5. Distributions of bias for three values of α for a 30% source wobble.

IV. CONCLUSIONS

We have presented a simple analytic model that demonstrates the possibility of a bias in the Opacity-on-Z experiments if the drive source is not uniform side-to-side and wobbles from shot-to-shot. The model is nothing more than manipulation of nonlinear functions, and a major counterpoint to this model is that not all opacities are overestimated on the experimental platform. Nevertheless, the model may indicate a possible source, or one of multiple possible sources, of the continuing overestimation of opacity on the Opacity-on-Z experimental platform. This source wobble bias could work in conjunction with the potential bias pointed out by Morris[3] since both are overestimations of the opacity. This analytic model shows that the bias, if real, could be highly variable across the lines—the peaks and valleys—of the spectral opacity as a function of radiation frequency, and presumably it would be worse for the valleys, which are weighted heavily when taking Rosseland averages. Some ways to address this bias, if it is real, are to employ additional statistical analyses, design new experimental setups, design less uniform foam cores that are (lower energy, but) maybe more predictable, and return to shooting and analyzing full-moon foils.

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