Sloped Bottom Tanks and Areal Density – Part II: Functional Behavior of Projected Areal Density

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INTRODUCTION

Part I (Ref. 1) of this topic introduces a practical case which led to a more fundamental question of physical behavior. That question was investigated in Part II.

Savannah River Site's H-Canyon Operations asked criticality safety for relief from the single parameter mass limit in the large volume sloped bottom decanters. This resulted in an investigation as to whether a fixed fissile mass calculated by a 0.40 g U-235/cm^2 areal density applied to the cross sectional area of the inner chamber of the decanter would remain safely subcritical under all expected concentration conditions. Part I of this work detailed that investigation and concluded that indeed the areal density based mass limits were safe for the two sizes of H-Canyon decanters, both of which have sloped bottoms. This was a practical, real world application, that will result in savings and efficiency.

However, this immediately brought up another, more fundamental question. The application of areal density limits as they are generally understood is to project the fissile mass of a system onto a surface. This areal density value is then comparable to an infinite slab of a certain thickness which is well understood and characterized by experimental data and well simulated by most modern computational programs. The underlying assumption is that the surface the mass is projected onto is a plane orthonormal to the remaining two dimensions of the system.

Since the decanters have sloped bottom heads, the areal density projection plane is not orthonormal to the other dimensions. This prompted further investigation into the more fundamental behavior of such systems. In particular, what does areal density mean in a system where the plane of projection may not be orthonormal to the remaining dimensions of the system? Under what conditions would areal density based mass limits no longer work? Is there a relationship between slope, area, and what may be called a *projected* areal density (PAD) where the plane of projection is not orthonormal to the other dimensions? Is there a conservative adjustment that could be made to the value of the PAD (or a PAD based mass limit) to account for sloped tanks? These questions are explored in this work.

Computational modeling was performed using KENO-VI in the SCALE 6.1 code package validated for site process applications in Ref. 2.

DESCRIPTION OF THE WORK

Background Theory

Basic nuclear criticality safety handbook data like that contained in Ref. 3 relates critical mass of a system to some controllable parameter such as the concentration of a fissile isotope in solution. This data may then translate into criticality safety limits like those prescribed in Ref. 4 which include the U-235 areal density limit in $^{235}UO_2(NO_3)_2$ solution of 0.40 g U-235/cm^2.

The areal density is a mathematical construct derived from the data. It is typically regarded as projecting the fissile mass of a system onto a plane orthonormal to the other two dimensions of the system such that it can be compared to the critical dimension of an infinite slab. In practice this is often an analog to spilling material on a floor or filling a flat bottom tank or tray. Along this line, the areal density is only applicable per se when the projection surface is flat.

In practice however, flat surfaces are not always economic, convenient, available, or safe from a chemical or processing hazard aspect. For example, it is not uncommon for a slab tank, collection basin, or glovebox to have some kind of safety drain or sump. Part I of this work showed that for a tank with a slightly sloped bottom, a mass limit based on areal density was still safe for the range of conditions experienced and due to the slight slope of the tank. To show this was true however, required significant calculational effort. It would be much more convenient and efficient if there was some known, understood relationship between areal density and the effect of sloped bottom tanks.

The objective of this investigation was to determine if a quantifiable, functional relationship exists between critical fissile mass (such as presented in Ref. 3) and projected areal density (PAD) which for the purpose of this work will be areal density projection of the fissile mass onto a plane wherein the plane of projection may or may not be orthonormal to the remaining dimensions of the system.

Analytical Approach

To closely parallel the nuclear data available in Ref. 3, all calculations are made assuming a solution of pure $^{235}UO_2(NO_3)_2$ with no excess nitric acid and having full reflection modeled by 60 cm of water in all directions followed by reflective boundary conditions. Cylinders of fissile solution, with no container walls assumed, were modeled at radii of 51.4 cm, 70.5 cm, 121.0 cm, and 150.5 cm which are the inner dimensions of the decanters from Part I and the inner dimensions of the 8 foot and 10 foot diameter H-Canyon tanks. The numerical precision is an artifact of converting inches to cm not an indicated of measurement tolerance. This effectively bounds the dimensions of most of the process vessels used in H-Canyon. The bottoms of these fissile solution cylinders were modeled with slopes between 0% (flat) and 15% using the KENO VI macro-body of a rotated wedge. The resulting general arrangement is shown in Fig. 1 for a 121.0 cm radius tank, solution height of 75 cm and a slope of 10%.

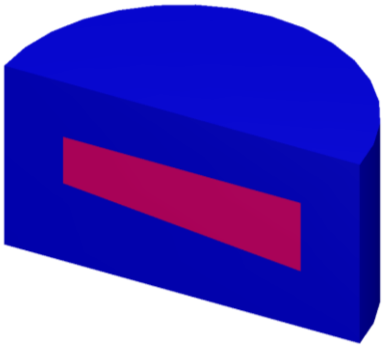

Fig. 1. Sloped bottom, water reflected fissile solution.

For a fixed slope and radius, the height of the solution is varied from as low as 6.35 cm up to 300 cm. At each height, the concentration of U-235 in the solution is increased until the model is calculated to be critical, i.e. until k-calculated is within 1.000 +/- 0.001 and the statistical uncertainty is less than 0.001 Δk. The fissile mass of the solution is then calculated from its volume and concentration. The mass may then be projected onto one of the non-orthonormal planes being investigated – i.e. onto either the sloped bottom surface of the tank or onto the flat surface of the solution to produce the PAD. For this work the PAD is selected to be projected onto the solution surface since this the dimension most easily defined on design drawings and understood by Operations and Engineering. The data could easily be renormalized to project onto the sloped tank bottom.

It was noted in Part I that once the solution volume drops below the upper or shallow end of the tank, the solution takes on the shape of a truncated circular wedge. Since the parameter being varied is the concentration and since the parameters of interest are mass and area, calculations of volume and area are necessary. When the solution is above the shallow end, it is simply the volume of a right circular cylinder above the shallow end plus one half the volume of right circular cylinder of a height equivalent to the rise from the deep to shallow end. Below the level of the shallow end, the solution area and volume are computed using integration calculations of the truncated circular wedge as detailed in Part I.

Once all PADs have been calculated for the range of heights for a given slope and radius, the minimum PAD is determined as would be done with experimental data to set a bounding safety parameter. Once all calculations are completed for all heights, slopes, and radii, the relationship between the minimum PADs and the parameters is examined.

RESULTS

For each of the radii examined, a figure similar to Fig. 2 may be constructed from the data. Fig. 2 illustrates the PAD which results in a critical configuration for various heights of solution. Each data series on Fig. 2 represents a different slope of the tank bottom and Fig. 2 is only for the 121.0 cm radius tank. The horizontal axis is the critical height of the solution as measured from the deep end of the tank. The minimum or limiting PAD, which is the lowest point on each curve, clearly shifts based on slope of bottom head. The smallest height for which a critical configuration can be achieved (i.e. the left most end points of each curve) also varies with the slope of bottom head. At that end point, the unusual truncated wedge shape, even with full water reflection, has sufficient neutron leakage out of the surface area of the solution such that no concentration of fissile material will go critical. Note, the limit of this analysis is the material remaining a homogenous solution, i.e. the concentration of uranium analyzed is kept less than the precipitation limit. For each radius of solution, the behavior followed the same trends but the values varied. Each curve on Fig. 2 represents a different slope and each slope has a unique minimum for the PAD.

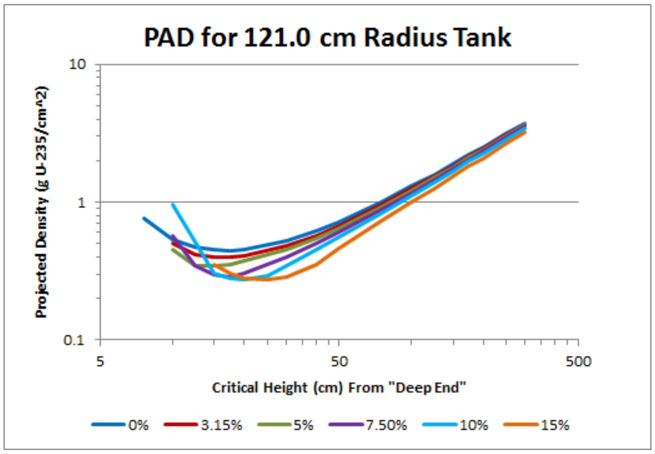

Fig. 2. PAD for the 121.0 cm radius tank at various slopes of the bottom head.

The minima for all of the cases analyzed are tabulated in Table I. Note that each datum is the minimum value from 15 to 20 data points in slope's PAD vs. height series. Since KENO VI does not have an automated criticality search feature on uranyl nitrate solution by atom density, each of the 15 to 20 slope data points is the result of manual iteration on concentration (to three decimal place resolution) taking between 5 and 15 iteration cases each. Therefore, this was not an insignificant effort to produce this relatively small amount of data.

The importance of analyzing the solution down into the partially filled truncated wedge shape of the bottom head can be seen in Table II. Table II lists the point in each tank at each slope at which the solution height, as measured from the deep end of the tank, breaks the plane of the shallow end of the tank. This is the point at which the solution shape goes from being a truncated wedge to a full wedge with a right circular cylinder atop it. The highlighted cells of Table II indicated tank designs wherein the minimum PAD occurred inside the partially filled truncated wedge.

Table I. Minimum PAD Results in g U-235 per cm^2.

Slope	51.4 cm radius	70.5 cm radius	121.0 cm radius	150.5 cm radius
0%	0.4919	0.4669	0.4456	0.4414
3.15%	0.4904	0.4588	0.4022	0.3677
5%	0.4868	0.4476	0.3507	0.2845
7.5%	0.4781	0.4219	0.2886	0.2651
10%	0.4685	0.3887	0.2753	0.2627
15%	0.4305	0.3335	0.2803	0.2713

Table II. Depth at Which Solution Breaks Plane of Shallow End of Tank in cm as Measured from Deep End*.

Slope	51.4 cm radius	70.5 cm radius	121.0 cm radius	150.5 cm radius
3.15%	3.24	4.44	7.62	9.48
5%	5.14	7.05	12.10	15.05
7.5%	7.72	10.57	18.15	22.57
10%	10.29	14.10	24.19	30.10
15%	15.43	21.15	36.29	45.15

*Highlighted cells are tank designs in which the minimum PAD occurs in the partially filled bottom head.

To aid in conceptualization, each value of PAD in Table I may be translated into its more common or tangible physical quantities. These quantities are provided for the reader's reference on Table III which presents for each minimum PAD case listed on Table I the associated total solution volume, total fissile mass, resulting fissile concentration, and resulting moderation ratio. As the reader can deduce, Table III data did not exhibit consistent functional behavior for the system like the minimum PAD values did.

The data from Table I is plotted in Fig. 3 where it may be observed that the minimum PAD follows a behavioral pattern. For the range of slope and radius investigated herein, this behavior is parabolic in both radius and slope. Radius is chosen as the parameter of interest here because most of the process vessels in the H-Canyon facility are circular. The data could easily be re-functionalized in terms of area, i.e. the data is also parabolic in terms of the square root of area.

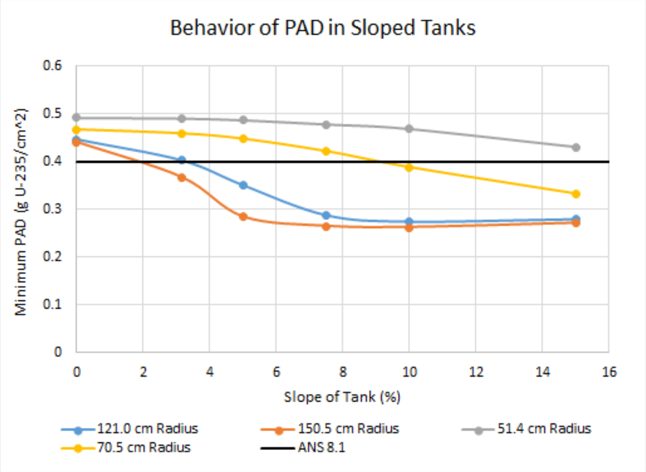

Fig. 3. Minimum PAD versus the slope for various radii.

For each of the data series plotted in Fig. 3, a parabolic fit was computed. Parabolic fitting was chosen due to its level of accuracy versus the complexity of the equation. Parabolic fitting of the curves (PAD as a function of slope) in Fig. 3 gave fit R^2 values between 0.957 and 0.999 even if some of the curves do not appear parabolic from observation. The behavior of the coefficients of those relationships is then also fitted to a parabolic function with respect to radius of the tank. Again, parabolic fitting limited the complexity of the equation while still returning fit R^2 values between 0.938 and 0.984. Therefore, reasonable agreement is expected from an equation with only nine coefficients.

Merging the fits results in the following functional form of minimum PAD in terms of % slope *s* and radius *r* in cm.

$$PAD = (6.199*10^{-8})s^2r^2 + (8.786*10^{-7})sr^2 + (2.126*10^{-6})r^2 + (9.071*10^{-6})s^2r – (5.674*10^{-4})sr – (8.537*10^{-4})r – (1.086*10^{-3})s^2 + (2.919*10^{-2})s + (5.262*10^{-1})$$

Fig. 3 also includes a solid line marked "ANS 8.1" for reader reference which is the 0.4 g U-235/cm^2 areal density limit from Ref. 3. The minimum PAD values for the slope and radius conditions in Table I are calculated using the relationship above and shown in Table IV. The agreement is acceptable but not always in the conservative direction.

CONCLUSIONS AND FUTURE WORK

Over the range of conditions analyzed it was found that the minimum PAD follows a behavior that can be characterized as parabolic in both radius and slope of the tank. One approach to applying this result is that if fissile solution is being stored in a tank with a sloped bottom, one could adjust down the ANS 8.1 single parameter areal density by this trend (function or data) and apply the lower PAD to the cross-sectional area of the tank in question. This could be done in lieu of extensive confirmatory calculations. If the user chooses to apply the functional relationship rather than the data, some small additional margin should be applied to ensure conservatism since agreement with data is acceptable but not always in the conservative direction.

Alternatively, if the geometry (slope and radius or area) of the tank are bounded by the available date, one could select a limiting PAD based on Table I, say 0.23 g U-235/cm^2.This would then be applied to the cross sectional area of the tank in question to generate a mass limit. The goal here being to prevent restricting the process to a single mass in an industrial scale vessel and to prevent having to do extensive calculations like those in Part I.

More and independent confirmation of this behavior and vetting of the approach would be necessary before this could be generally applied a means of setting a criticality safety limit on fissile mass. For future work, it should be investigated if the behavior holds for the generic parameter of area rather than radius by examining non-circular tanks and deriving the PAD from projection areas in those geometries.

REFERENCES

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2. S. H. Finfrock, et. al, “SCALE 6.1 Validation for SRNS Personal Computers,” N-CLC-G-00166, Savannah River Nuclear Solutions, (2016).
3. H.C. Paxton and N.L. Pruvost, “Critical Dimensions of Systems Containing ^{235}U, ^{239}Pu, and ^{233}U, 1986 Revision”, LA-10860-MS, Los Alamos National Laboratory, 1986.
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Table III. Volume, Mass, Moderating Ratio and Concentration for Minimum PAD Cases.

Slope	Volume (cm^3)	Mass (g U-235)	H/U-235	Conc. (g U-235/L)
51.4 cm radius				
0%	145447.2	4088.08	909.751	28.107
3.15%	131983.0	4075.63	826.511	30.880
5%	144850.9	4046.26	915.491	27.934
7.5%	134163.6	3973.66	862.462	29.618
10%	123476.4	3893.70	809.013	31.534
15%	81323.7	3578.24	574.933	44.000
70.5 cm radius				
0%	273137.5	7287.85	958.086	26.682
3.15%	238488.1	7160.13	850.594	30.023
5%	218131.5	6985.44	796.371	32.024
7.5%	190628.5	6584.88	737.044	34.543
10%	163125.5	6066.96	683.321	37.192
15%	147290.5	5095.37	735.931	34.594
121.0 cm radius				
0%	804498.7	20484.95	1006.000	25.463
3.15%	629348.2	18490.25	869.587	29.380
5%	411518.3	16121.64	647.844	39.176
7.5%	387554.7	13119.50	752.439	33.852
10%	372410.2	11188.32	850.018	30.043
15%	372014.5	9465.54	1006.770	25.444
150.5 cm radius				
0%	1245179.8	31404.68	1015.820	25.221
3.15%	730031.8	26160.69	709.849	35.835
5%	531888.0	20236.21	667.597	38.046
7.5%	466893.0	15692.27	757.981	33.610
10%	440000.0	13189.00	851.985	29.975
15%	433000.0	10970.06	1011.170	25.335

Table IV. Calculated Minimum PAD in g U-235 per cm^2.

Slope	51.435 cm radius	70.485 cm radius	120.9675 cm radius	150.495 cm radius
0%	0.4879	0.4766	0.4540	0.4459
3.15%	0.4907	0.4549	0.3794	0.3482
5%	0.4882	0.4409	0.3440	0.3064
7.5%	0.4798	0.4205	0.3063	0.2683
10%	0.4657	0.3983	0.2800	0.2512
15%	0.4204	0.3488	0.2618	0.2801