Accurate Resonance Calculation of the Fuel Rod with Non-Uniform Temperature Profile

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INTRODUCTION

It is difficult to obtain a rigorous result of resonance self-shielding considering the non-uniform temperature profile inside the fuel rod using ultra-fine-group (UFG) slowing down method based on collision probabilities (CP). For the uniform temperature problem, CP of the cylindrical geometry can be obtained rapidly by interpolation on a pre-tabulated CP table. However, for the non-uniform temperature problem, CP are affected by the macroscopic cross section of each ring with different temperature, thus CP of the non-uniform temperature problem can only be calculated dynamically, which is very time consuming.

In previous research activities [1], an approximation is used to reduce the computational time of calculating CP. Instead of calculating CP dynamically, the first-flight CP from region *i* to region *j* is obtained from a uniform temperature problem with the same temperature as that in region *i*. It was reported that the approximation is accurate for resonance calculation and k-infinity evaluation [1]. However, in this study, we find the inconsistency in this method and it is proved theoretically and numerically. Based on the analysis, we also proposed another approach which is more accurate based on the two-dimensional CP table and temperature perturbation technique. Preliminary results show that the approach is suitable for accurate resonance calculation for non-uniform temperature problems.

THEORY

In typical resonance calculation code based on UFG slowing down method, the slowing down equation is solved as:

$$V_i\Sigma_{ti}\left(E\right)\phi_i\left(E\right)=\sum_{j=1}^{N+1}V_jP_{ji}\left(E\right)Q_j\left(E\right) \tag{1}$$

Applying the narrow resonance approximation on the non-resonant material and the reciprocity theorem, the slowing down equation is rewritten as:

$$\phi_i\left(E\right)=\sum_{j\in f}\frac{P_{i,j}\left(E\right)}{\Sigma_{t,j}\left(E\right)}Q_j\left(E\right)+\sum_{k\in m}P_{i,k}\left(E\right) \tag{2}$$

To solve the equation, Pij needs to be evaluated for each sub-region of the pin. For the cylindrical geometry, Pij can be calculated by ray tracing and integration as in SRAC code [2]. In practice, Pij is pre-calculated for different macroscopic cross sections of the resonant absorber and Pij in the target pin cell is obtained by interpolation. In the PEACO routine of the SRAC code [3], there is a limitation that only one or two resonant absorbers can be considered for setting up the interpolation table. The limitation is due to the size limit of the table because for more resonant absorbers, multi-dimensional table is needed, which is cumbersome to be prepared.

However, for the fuel rod with non-uniform temperature profile, the problem contains multiple resonant absorbers because resonances of the same isotope have different representations in different temperatures due to Doppler broadening. To reduce the time for computing P_{ij}, a flat source assumption is used widely for non-uniform temperature problems. The flat source assumption was first used for the Spatially Dependent Dancoff Method in PARAGON for rim effect, assuming that the composition variation has almost no effect on P_{ij}. Then it is extended to the non-uniform temperature cases, assuming that the blackness of the target ring is obtained in the uniform temperature cylinder with the same temperature as in the target ring [4]. In MPACT code [1], similar approximation is proposed that

$$P_{i,j}\left(E\right)=P_{i,j}^{T_i}\left(E\right) \tag{3}$$

which means that Pij can be evaluated in the special case with uniform temperature of Ti.

The flat source assumption neglects the flux tilt caused by non-uniform macroscopic cross sections of each ring. It is valid in the rim effect of the burnup scenario because macroscopic cross sections do not change much during burnup. However, in a non-uniform temperature case, the flux tilt caused by non-uniform macroscopic cross sections is more severe because Doppler broadening has considerable impact on resonances of ^{238}U, which is the main contributor of the fuel material.

A theoretical proof is that the approximation in Eq. (3) is inconsistent with the reciprocity theorem. Supposing a 10-ring fuel rod with equal volume, we have:

$$P_{1,10}\left(E\right)=P_{1,10}^{T_1}\left(E\right)\,\&\,P_{10,1}\left(E\right)=P_{10,1}^{T_{10}}\left(E\right) \tag{4}$$

in reciprocity theorem:

$$P_{1,10}\left(E\right)\Sigma_{t,1}\left(E\right)=P_{10,1}\left(E\right)\Sigma_{t,10}\left(E\right) \tag{5}$$

It is easy to figure out that there is contradictory between Eq. (4) and Eq. (5) because the approximation in Eq. (4) does not fit the reciprocity theorem.

To save space, the numerical proof that flat source assumption causes errors is given in the next section along with the new proposed approach. To consider the flux tilt caused by non-uniform macroscopic cross sections, a

temperature perturbation technique is proposed to solve the problem. The microscopic self-shielding cross section of the ring *i* in the non-uniform temperature case can be approximated as:

$$\sigma_i \approx \prod_{i=1,10}\left(\frac{\sigma_i^{uniform\to T_i}}{\sigma_i^{uniform}}\right)\cdot\sigma_i^{uniform} \quad (6)$$

In Eq. (6), $\sigma_i^{uniform}$ is the cross section of ring i calculated by UFG method in a uniform temperature rod. $\sigma_i^{uniform\to T_i}$ is the new cross section calculated supposing that only the temperature of ring i is changed to the target temperature. The ratio of $\sigma_i^{uniform\to T_i}/\sigma_i^{uniform}$ is the impact on the cross section from temperature perturbation in each fuel ring. The product of all the ratio can be approximated as the final correction on the cross section obtained in uniform temperature. Eq. (6) is not based on rigorous derivation while it is a reasonable approximation to address the flux tilt in non-uniform temperature problems by decomposing the perturbation effect from temperature difference.

It is noticed that in the method, the UFG code has to be able to treat two resonant absorbers, which is feasible using the methodology in PEACO [3]. In PEACO, a two-dimensional CP table is pre-calculated considering two types of resonance absorber. It brings a modest increase on computational costs. The calculation procedure is as follows:
(1) The UFG calculation is first performed on the same fuel rod with the uniform temperature. The uniform temperature used can be calculated by volume average.
(2) Change the value of temperature ring i to the correspondent temperature in the non-uniform profile. The UFG calculation is performed again and the ratio of cross section change is recorded.
(3) Perform step (2) on all fuel rings. Using Eq. (6) to obtain the final cross section of each ring.
Using the above calculation procedure, the flux tilt caused by non-uniform temperature can be approximately considered.

RESULTS

In this section, the UO_2 fuel pin cell extracted from JAERI LWR benchmark is mainly studied [5]. The fuel region is divided to 10 rings with equal volume (No.1 as center region and No.10 as outmost ring). An in-house UFG slowing down solver code based on CP is used to obtain effective cross sections. The code is capable of treating two resonant absorbers at one time. UFG slowing down equation is solved from 9118 eV to 4 eV and cross section is condensed to 13 groups as in WIMS-69 group structure [6]. The reference results is from Monte Carlo code with libraries generated with specific temperatures and microscopic absorption cross section of ^{238}U is mainly compared. The temperature profile is from Ref. 4 and 997K is used in the temperature perturbation technique as the uniform temperature.

Fig.1 shows relative errors of group 20 to 25 and group 27 in region No.1 and No.3. These groups are selected due to large resonances. Fig.2 shows the relative errors of group 23 and 25 in each ring along the radius.

Fig.1 and Fig.2 show that the resonance capture of inner region of the fuel rod is underestimated by the flat source assumption. According to the study of spatially self-shielding [1,4], neutrons are mostly absorbed at outmost ring. When evaluating the self-shielding of inner most ring, the outmost ring has the cross section at 1190K, larger than the real case of 820K, leading to overestimated resonance capture of the outmost ring, which causes lower spectrum for inner rings. Thus the cross section of inner rings are underestimated. On the other hand, the temperature perturbation technique improve the accuracy of cross section prediction by considering the impact on spectrum from the non-uniform temperature profile without calculating CP dynamically.

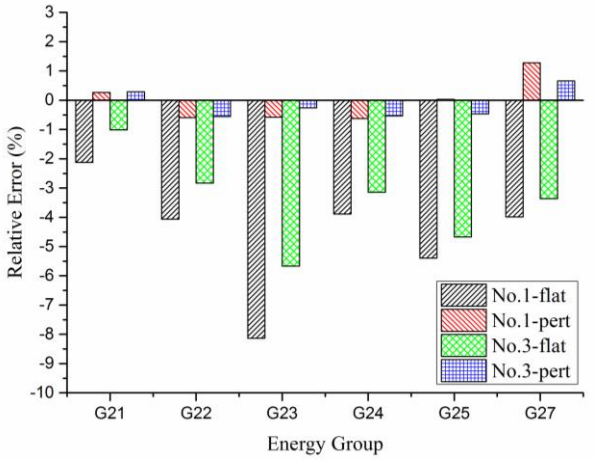

Fig. 1. Relative cross section errors in region No.1 and No.3 by two approaches (flat: flat source assumption. pert: temperature perturbation technique.).

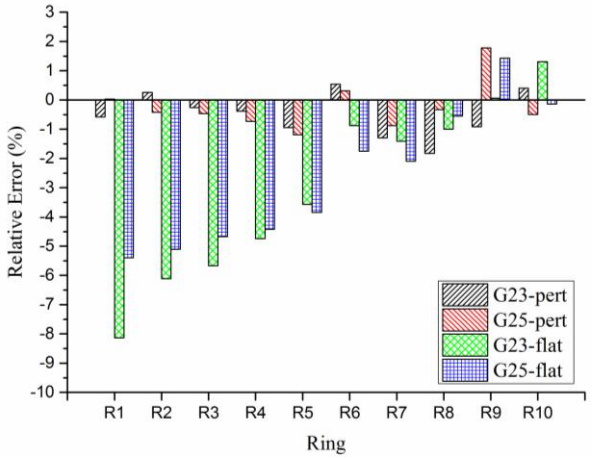

Fig. 2. Relative cross section errors in each ring

The cross section prediction affects the intra-pellet reaction rate distribution. Fig. 3 shows errors of the absolute microscopic absorption reaction rate of ^{238}U at group 27.

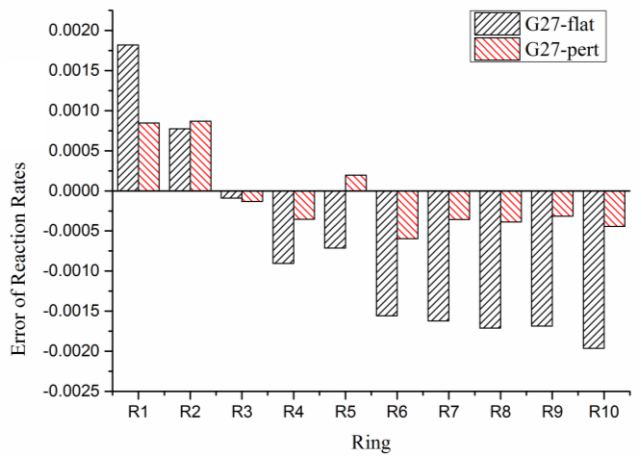

Fig. 3 Errors of reaction rates distribution.

Results indicates that there exists error cancelation in the intra-pellet reaction rate along the radius. However, although the flux in inner regions are lower, deviations brought by flux source assumption still has a non-negligible impact on the reaction rate calculation for whole pin. Using the self-shielded cross section as input, k-infinity is evaluated for the pin cell by an in-house MOC code and non-resonant cross sections are directly read from the standard multi-group library from WLUP [6]. TABLE I shows the k-infinity error caused by self-shielding calculation. The fuel of original pin cell is 6.5 wt% enrichment and we also tested another case of 1.5 wt% enrichment. Results show that the k-infinity of flat source assumption is overestimated and the new approach improves the accuracy of k-infinity evaluation considering the spatially self-shielding.

TABLE I. k-infinity error comparison (pcm)

	k_{inf}-flat	k_{inf}-pert
6.5 wt% enrichment	310.4	86.4
1.5 wt% enrichment	197.3	-48.3

DISCUSSIONS

In the routine lattice physics calculations, the scenario of non-uniform temperature input is rare. However, this work will benefit the accuracy in following cases:

1. A tight coupling of lattice physics code and fuel performance code on temperature and power distribution.
2. Evaluation of effective uniform temperature of the whole rod which preserves the reaction rate of the case with non-uniform temperature profile.

The computational effort of temperature perturbation technique is larger than the flat source assumption. However, in practice, the impact ratio in Eq. (6) can be pre-calculated and tabulated for interpolation. The detailed analysis is undergoing.

CONCLUSIONS

In this study, a new approach is proposed for resonance self-shielding in the fuel rod with non-uniform temperature profile. Preliminary analysis shows that the widely used approach based on flat source assumption will bring non-negligible deviations on the cross section prediction for inner rings of the pin and the new approach based on the temperature perturbation technique is able to improve the accuracy without on-line CP calculations.

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