Estimation of Subcriticality Using Particle Filter Method

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INTRODUCTION

Subcriticality estimation is carried out by the particle filter method, which is one of the data assimilation methods, using a time series data of neutron count.

For the decommissioning of the Fukushima Daiichi (1F) Nuclear Power Station (NPS), various R&D activities have been conducted [1]. The removal of fuel debris is one of the challenging issues since we never have an experience of the retrieval of fuel debris in Japan and the conditions inside reactors of Units 1-3 of 1F-NPS are still unknown. If the retrieval of fuel debris is conducted under the submersion condition, the moderation ratio, or H/U ratio, could change because of the submerging and the drilling operation. The variation of H/U ratio could cause unexpected insertion of positive reactivity. Thus, approach to the critical condition has to be detected as soon as possible and the subcritical state kept using neutron absorbers, in order to prevent undesirable exposures for workers and to reduce radiation influences on the environment and the general public around 1F-NPS.

One of the detection methods for the criticality approach is subcritical monitoring using the Inverse Kinetics Method (IKM). Considering the utilization of IKM in the 1F-NPS, it is noted that not only the subcriticality $(-\rho)$ but also the point kinetics parameters $(\Lambda, \beta_{\mathrm{eff}})$ and neutron source strength can change simultaneously when H/U ratio changes. The subcriticality cannot be appropriately estimated when the adequate point kinetics parameters are not used. In order to address this issue, estimation of subcriticality is carried out by utilizing the particle filter method [2].

In the present summary, a feasibility study is conducted for the subcriticality estimation using the particle filter method. The rest part of this summary is organized as follows. First, the methodology of the particle filter method is explained. Next, the estimated result of subcriticality is shown for an experimental result at the Kindai University reactor (UTR-KINKI) [3]. Finally, the concluded remarks are summarized.

METHODOLOGY

Point Kinetics Equation

In this section, the subcriticality estimation method based on the particle filter method is explained. In order to compare estimated results of subcriticality, the conventional Inverse Kinetics Method (IKM) was also used [4]. Both the particle filter method and IKM are based on the point kinetics equation [5]. The point kinetics equation with six delayed neutron groups and external neutron source can be described as:

$$\frac{dn(t)}{dt} = \frac{\rho(t) - \beta_{\mathrm{eff}}}{\Lambda} n(t) + \sum_{k=1}^{6} \lambda_k C_k(t) + S, \quad (1)$$

$$\frac{dC_k(t)}{dt} = \frac{\beta_k}{\Lambda} n(t) - \lambda_k C_k(t), \quad (2)$$

where

n : number of neutrons;
C_k : number of the k-th group delayed neutron precursors;
ρ : reactivity, and subcriticality corresponds to $-\rho$;
β_{eff} :effective delayed neutron fraction, $\beta_{\mathrm{eff}} \equiv \sum_{k=1}^{6} \beta_k$;
β_k :delayed neutron fraction for the k-th delayed neutron precursor group;
λ_k : decay constant for the k-th delayed neutron precursor group;
S : external neutron source strength.

When the time-derivative term of Eq. (2) is equal to zero, the number of delayed neutron precursors in the initial steady state can be obtained as Eq. (3):

$$C_{k,0} = \frac{\beta_k}{\lambda_k \Lambda} n_0, \quad (3)$$

where the subscript 0 means the value at the initial steady state condition.

Inverse Kinetics Method

For comparison with the estimated subcriticality using the particle filter method, the subcriticality was estimated by the conventional IKM with an external neutron source [4]. Here, the prompt jump approximation was assumed for simplicity:

$$\Lambda \frac{dn(t)}{dt} \approx 0. \quad (4)$$

By substituting Eq. (4) into Eq. (1), the following equation can be obtained:

$$\rho(t) = \beta_{\mathrm{eff}} - \frac{\Lambda}{n(t)} \left(\sum_{k=1}^{6} \lambda_k C_k(t) + S \right), \quad (5)$$

where $C_k(t)$ can be obtained by numerically solving Eq. (2) using measurement value of $n(t)$. Therefore, the real-time monitoring of $\rho(t)$ is achieved by measuring time-series data of $n(t)$ using a nuclear instrumentation system.

Particle Filter Method

The particle filter (PF) method is one of the method for non-linear dynamic system estimation [2]. In the PF method, many particles are used to approximately express a probability distribution of state variables (*e.g.* number of neutrons and reactivity).

In order to estimate subcriticality using using PF method, the point kinetics equation should be described as the state space model. The state space model consists of the following two models, the system model and the measurement model.

The PF algorithm consists of two steps: one is the "prediction," and another the "update." The prediction is a temporal update for a vector of state variables (a state vector) using the system model. The update is an estimation of the state vector using the likelihood, which is evaluated by comparing the predicted state vector with an actual measurement value. Here, the measurement model is utilized for this comparison.

In this section, the state space model of this study and the PF algorithm are shown. First, the system model in this study is explained. The reactivity ρ is assumed as a linear function with respect to time-variable [6]:

$$\rho(t) = \alpha + \omega t, \tag{6}$$

where α and ω express coefficients to simulate step- and ramp-wise reactivity variations, respectively. By differentiating Eq. (6) with time-variable, the following time differential equation for ρ is obtained:

$$\frac{d\rho}{dt} = \omega, \tag{7}$$

where the time derivative of ω is assumed to be zero:

$$\frac{d\omega}{dt} = 0. \tag{8}$$

The full-implicit method is applied to obtain the finite difference equations for Eqs. (1), (2), (7) and (8). By the backward difference approximation, the difference equations can be expressed as:

$$n_{t+1} = \frac{\frac{1}{\Delta t} n_t + \sum_{k=1}^{6} \lambda_k \mu_k C_{k,t} + S}{\frac{1}{\Delta t} - \frac{\rho_{t+1} - \beta_{\mathrm{eff}}}{\Lambda} - \sum_{k=1}^{6} \lambda_k \xi_k}, \tag{9}$$

$$C_{k,t+1} = \xi_k n_{t+1} + \mu_k C_{k,t}, \tag{10}$$

$$\rho_{t+1} = \rho_t + \omega_{t+1} \Delta t, \tag{11}$$

$$\omega_{t+1} = \omega_t, \tag{12}$$

where Δt is sampling interval; and the subscript t means time step; and μ_k and ξ_k are respectively defined as:

$$\mu_k \equiv \frac{1}{1 + \lambda_k \Delta t}, \tag{13}$$

$$\xi_k \equiv \frac{\beta_k \Delta t}{\Lambda(1 + \lambda_k \Delta t)}. \tag{14}$$

Here, the state vector $\boldsymbol{x}$ is defined as:

$$\boldsymbol{x} = [n \quad C_1 \quad C_2 \quad C_3 \quad C_4 \quad C_5 \quad C_6 \quad \rho \quad \omega]^T. \tag{15}$$

From Eqs. (9), (10), (11), and (12), $\boldsymbol{x}_{t+1}$ can be expressed by a functional form, *i.e.*, $\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t)$. By adding a noise term, the system model is obtained as:

$$\boldsymbol{x}_{t+1} = f(\boldsymbol{x}_t + \boldsymbol{v}_t), \tag{16}$$

where $\boldsymbol{v}$ is the system noise, which represents an uncertainty of simulation model, *e.g.*, numerical error and uncertainty due to stochastic process in the nuclear reaction.

Now, let us introduce the measurement model to express the relationship between the state vector of Eq. (15) and a measurement value:

$$\boldsymbol{y}_t = \boldsymbol{H}_t \boldsymbol{x}_t + \boldsymbol{w}_t, \tag{17}$$

where

$\boldsymbol{y}_t$:measurement vector,
$\boldsymbol{H}_t$:measurement matrix,
$\boldsymbol{w}_t$:measurement noise.

In this study, it is assumed that only the number of neutron $n(t)$ is measurable. Thus $\boldsymbol{H}_t$ is defined by:

$$\boldsymbol{H}_t = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T. \tag{18}$$

Consequently, $\boldsymbol{y}_t$ and $\boldsymbol{w}_t$ correspond to scalar values of $n(t)$ and the statistical noise, respectively.

Next, the PF algorithm is summarized below:

0. Generate the *i*-th initial particle of state vector $\boldsymbol{x}_{0|0}^i$, according to the initial distribution $p(\boldsymbol{x}_0)$ $(1 \le i \le N)$:

$$\left\{\boldsymbol{x}_{0|0}^i\right\}_{i=1}^{N} \sim p(\boldsymbol{x}_0), \tag{19}$$

where N is the total number of particles; and p means a probability distribution; and $\boldsymbol{x}_{0|0}^i =$ $\left[n_0^i \quad C_{1,0}^i \quad C_{2,0}^i \quad C_{3,0}^i \quad C_{4,0}^i \quad C_{5,0}^i \quad C_{6,0}^i \quad \rho_0^i \quad \omega_0^i\right]^T$.

The PF algorithm is summarized as follows. For each step, repeat the procedure from $i = 1$ to $i = N$.

[Prediction]

1. Generate the system noise $\boldsymbol{v}_{t-1}^i$.
2. Predict the *i*-th particle $\boldsymbol{x}_{t|t-1}^i$ using Eq. (16):

$$\boldsymbol{x}_{t|t-1}^i = \boldsymbol{f}\left(\boldsymbol{x}_{t-1|t-1}^i + \boldsymbol{v}_{t-1}^i\right), \tag{20}$$

where the subscript $t|t-1$ means a priori particle at the time step t derived from a posteriori particle $\boldsymbol{x}_{t-1|t-1}^i$ at the time step $t-1$.

[Update]

3. Evaluate the likelihood l_t^i of the i-th particle: Using the actual measurement value y_t, the likelihood of the i-th particle $\boldsymbol{x}_{t|t-1}^i$ is calculated. In this study, the likelihood l_t^i is assumed to follow the Gaussian distribution, *i.e.*, l_t^i is calculated as:

$$l_t^i = \frac{1}{\sqrt{2\pi R_t}} \exp\left(-\frac{\left(y_t - \boldsymbol{H}_t \boldsymbol{x}_{t|t-1}^i\right)^2}{R_t}\right), \tag{21}$$

where R_t is the variance of the measurement noise.

4. Compute the weight of the i-th particle:

$$w_t^i = \frac{l_t^i}{\sum_{i=1}^{N} l_t^i}. \tag{22}$$

5. Resample a posteriori particle $\boldsymbol{x}_{t|t}^{i}$ at the time step t : By resampling particles on the basis of the weight w_t^i, the degeneracy problem of particles can be mitigated. Namely, more particles having larger weights are resampled, and particles having nearly zero weights are eliminated. After this resampling, the weight w_t^i is set to $w_t^i = 1/N$.
6. Compute the weighted average as the expected value:

$$\bar{\boldsymbol{x}}_t = \sum_{i=1}^{N} w_t^i \boldsymbol{x}_{t|t}^i = \frac{1}{N}\sum_{i=1}^{N} \boldsymbol{x}_{t|t}^i. \tag{23}$$

In addition, the weighted standard deviation is computed as the uncertainty (1σ).

$$\sigma_t = \sqrt{\sum_{i=1}^{N} w_t^i \left(\boldsymbol{x}_{t|t}^i - \bar{\boldsymbol{x}}_t\right)^2} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} \left(\boldsymbol{x}_{t|t}^i - \bar{\boldsymbol{x}}_t\right)^2}. \tag{24}$$

7. Steps 1-6 are repeated to monitor the temporal change of reactivity.

EXPERIMENTAL ANALYSIS

In order to validate the applicability of the subcriticality estimation using the PF method, the analysis was carried out by utilizing experimental data at the University Teaching and Research Reactor of Kindai University (UTR-KINKI) [3]. Time series data of neutron count were acquired at Kindai University's Advanced Nuclear Reactor Experiment 2017[3]. Neutron counts per 0.5 [s] were measured using the fission chamber. The reactor condition was a critical state without external neutron source. Point kinetics parameters for UTR-KINKI are shown in TABLE I [3].

TABLE I. Point kinetics parameters of UTR-KINKI[3]

Group k	Decay constant $\lambda_k[\mathrm{s}^{-1}]$	Delayed neutron fraction $\beta_k[-]$
1	0.0124	0.000305
2	0.0305	0.00164
3	0.111	0.00150
4	0.301	0.00324
5	1.14	0.00102
6	3.01	0.000207
Total		0.007912

Λ: 1.50×10^{-4} [s]

Calculation Condition of Particle Filter Method

In this section, the calculation conditions of PF method is explained. The total number of particles N was 1000. The time step Δt was 0.5 [s], which is the same as the time interval of measurement data. The external neutron source was negligibly small, *i.e.*, $S \approx 0$ [s^{-1}]. The point kinetics parameters of TABLE I were utilized in the PF method. In this study, the measurable state variable is the only neutron count. The i-th initial neutron count n_0^i was assumed as the measurement value at $t = 0$, that is, $n_0^i = y_0$. Using Eq. (3), the i-th initial delayed neutron precursor $C_{k,0}^i$ was obtained as:

$$C_{k,0}^i = \frac{\beta_k}{\Lambda\lambda_k} n_0^i = \frac{\beta_k}{\Lambda\lambda_k} y_0 = C_{k,0}. \tag{25}$$

The i-th initial reactivity ρ_0^i was assumed as a normal random number which follows $\mathcal{N}(0,10^{-6})$. Here, $\mathcal{N}(0,10^{-6})$ is Gaussian distribution which has mean 0 and variance 10^{-6}. In addition, it was assumed that $\omega_0^i \approx 0$. Namely, the initial particles are different reactivity but other state parameters are the same:

$$\left[\boldsymbol{x}_{0|0}^1 \ \cdots \ \boldsymbol{x}_{0|0}^N\right] = \begin{bmatrix} \begin{bmatrix} n_0^1 \\ C_{1,0}^1 \\ \vdots \\ C_{6,0}^1 \\ \rho_0^1 \\ \omega_0^1 \end{bmatrix} & \cdots & \begin{bmatrix} n_0^N \\ C_{1,0}^N \\ \vdots \\ C_{6,0}^N \\ \rho_0^N \\ \omega_0^N \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} y_0 \\ C_{1,0} \\ \vdots \\ C_{6,0} \\ \rho_0^1 \\ 0 \end{bmatrix} & \cdots & \begin{bmatrix} y_0 \\ C_{1,0} \\ \vdots \\ C_{6,0} \\ \rho_0^N \\ 0 \end{bmatrix} \end{bmatrix}. \tag{26}$$

In this study, the system noise $\boldsymbol{v}$ at each time step was given based on the Gaussian noise, *i.e.*, $\boldsymbol{v}_t^i$ is defined as:

$$\boldsymbol{v}_t^i = \begin{bmatrix} \eta_{1,t}^i & 0 & 0 & 0 & 0 & 0 & 0 & \eta_{2,t}^i & \eta_{3,t}^i \end{bmatrix}^T, \tag{27}$$

where $\eta_{1,t}$, $\eta_{2,t}$ and $\eta_{3,t}$ were assumed as a normal random number which follows $\mathcal{N}(0, y_t)$, $\mathcal{N}(0,10^{-10}|y_t - y_{t-1}|)$ and $\mathcal{N}(0, 10^{-10})$, respectively. The system noise is set on the basis of the following reason. The absolute difference of the measurement value $|y_t - y_{t-1}|$ becomes large due to rapid variation of neutron counts for a step reactivity change. Therefore, the variance of the system noise of the reactivity ρ_t is assumed proportional to $|y_t - y_{t-1}|$.

Estimation results

As the result of the PF, Figs. 1 and 2 show the estimated neutron count and reactivity, respectively. In Figs. 1 and 2, "Measurement" corresponds to the measured time series data of neutron count rate; "IKM" and "PF" are estimated values of reactivity using IKM and PF, respectively; "PF $\pm 2\sigma$" indicates the interval of $\pm 2\sigma$, where σ indicates the weighted standard deviation.

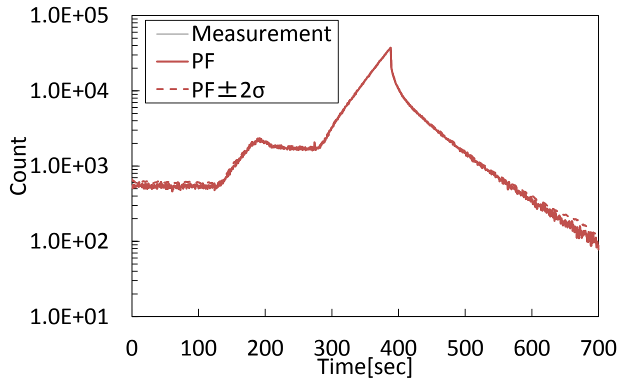

Fig. 1 Estimated neutron count

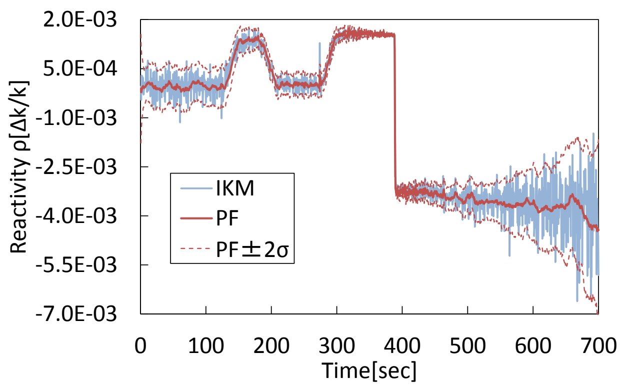

Fig. 2 Estimated reactivity

As shown in Fig. 2, it is confirmed that the estimated result of PF is consistent with the reference values of IKM, within the range of $\pm 2\sigma$.

CONCLUSION

In this summary, the applicability of the PF method to estimate the subcriticality was investigated. The estimated reactivity using the PF method is almost the same as that using IKM within the range of $\pm 2\sigma$.

In case of the retrieval of fuel debris at 1F-NPS, it is supposed that the temporal variations of $(\rho, \Lambda, \beta_{\text{eff}})$ could be occurred at the same time by changing H/U ratio due to the submersing and the drilling operations. As one of the future studies, the applicability of the PF method for the simultaneous estimation not only $(-\rho)$ but also $(\Lambda, \beta_{\text{eff}})$ will be investigated.

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