Application of the Ratio Correction Fission Matrix Method to a 2-D Four-Assembly Model

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INTRODUCTION

The Monte Carlo criticality calculation has gained popularity recently due to the rapid growth of the computational abilities. The Monte Carlo criticality simulation is accurate by using the exact geometry and continuous energy. However, the traditional Monte Carlo calculation requires very high computational costs to achieve a desired tally uncertainty for the detailed full core power distribution calculations. In addition, the source convergence is another problem of the traditional Monte Carlo calculation. It can be hard to tell when the source converges because of the slow convergence. Moreover, the traditional Monte Carlo calculation is only able to calculate the fundamental eigenvector and eigenvalue of the system. To solve the problems, the fission matrix method [1] has been introduced as an acceleration method of the traditional Monte Carlo criticality calculation.

The RAPID code (Real-time Analysis for Particle-transport and in-situ Detection) which originally designed for spent fuel pool criticality problem [2]–[4] has been applied on reactor physics problems [5]. The RAPID code is based on a fission matrix combination method [6] (which has recently been used in other work [7], [8]) and is able to rapidly solve the criticality problems. It accomplishes this by pre-calculating a small portion of the full fission matrix under a variety of conditions, and combining these to estimate a fine-scale, full-core fission matrix. However, the fission matrix combination method introduces errors especially with a large material enrichment discontinuity (e.g. neighboring assembly enrichment). To reduce the errors, a ratio correction method [9] has been proposed based on the original fission matrix combination method. The ratio correction fission matrix method has been tested on a two assembly model varying only in one dimension. In this paper, the method will be extended to a four-assembly model varying in two dimensions. It includes techniques and an assumption to estimate the correction ratios. Similarly, the ratio correction method can be applied to the whole core calculations based on the techniques introduced in the four-assembly model, which is the ultimate goal of this work.

RATIO CORRECTION DESCRIPTION

The criticality problem can be expressed in fission matrix form as

$$\vec{F} = \frac{1}{k} \cdot \boldsymbol{A} \cdot \vec{F} \tag{1}$$

Where $\vec{F}$ is the source distribution and k is the multiplication factor. The fission matrix elements $\boldsymbol{A}_{ij}$ represent the number of fission neutrons born in cell i per source neutron from cell j. From Equation (1), it can be inferred that the fundamental eigenvalue of the fission matrix $\boldsymbol{A}$ equals to the multiplication factor and the fundamental eigenvector represents the source distribution of the system.

The original fission matrix combination method assumes the neutron production in cell j with the inducing neutron from cell i depends only on the property of cell j. It implies that the fission matrix element $\boldsymbol{A}_{ij}$ does not change if the enrichment of the cell j stays the same. In this way, if a fission matrix is calculated independently for two models with different enrichments, then the true fission matrix for a mixed enrichment model can be estimated with the combination of the two original fission matrices.

However, the assumption in the original fission matrix combination method introduces errors at the material discontinuity due to the different flux spectrum. To reduce the errors, the ratio correction method has been proposed. That is, the original combined fission matrix is multiplied by a diagonal ratio correction matrix. It serves to multiply each row in the original combined fission matrix by a constant ratio. In a two assembly model, the pin-wise ratios have been proved to be linearly dependent on the enrichment of the other assembly [9]. It implies that the correction ratios can be interpolated with a ratio curve, which saves the Monte Carlo calculations to derive the ratios. However, if an assembly is surrounded by several different enrichment assemblies, the ratios cannot be directly interpolated by a ratio curve from any assembly. This paper will demonstrate techniques to estimate the ratios in an assembly surrounded by different assemblies.

NUMERICAL MODEL

The numerical model used is a 2-D 2×2 assembly section of the BEAVRS hot-zero-power benchmark [10], where each assembly has a uniform enrichment as presented in Fig. 1. Each assembly consists of 17×17 cells with 264 fuel pins and 25 guide tubes.

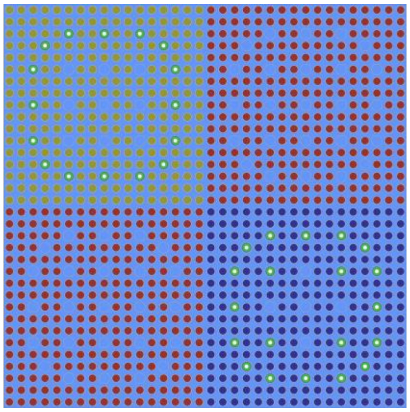


Fig. 1. The four-assembly numerical model.

There are three enrichment variations in the model including $1.6\ wt\%\ ^{235}U/U_{total}$, (red), $2.4\ wt\%\ ^{235}U/U_{total}$ (yellow) and $3.1\ wt\%\ ^{235}U/U_{total}$ (blue). For simplicity, only fresh fuels are considered in current work. The moderator is water at $0.74\ g/cm^3$ with 975 ppm boron. There are 20 burnable absorber rods in the $3.1\ wt\%\ ^{235}U/U_{total}$ enriched assembly and 16 burnable absorber rods in the $2.4\ wt\%\ ^{235}U/U_{total}$ enriched assembly. The periodic boundary conditions are used. The dimensions of the assembly model are summarized in Table I. The materials and dimensions are identical to the BEAVRS benchmark.

TABLE I. Dimensions of the assembly model

Parameter	Value [cm]
Fuel Rod Pitch	1.25984
Fuel Rod Inner Diameter	0.78436
Fuel Rod Outer Diameter	0.91440
Guide Tube Inner Diameter	1.12268
Guide Tube Outer Diameter	1.20396

RATIO DERIVATION

A technique to estimate the ratios in an assembly surrounded by different assemblies will be demonstrated. As shown in Fig. 2, four different models have been simulated by Serpent 2 [11]. The four models have the same material properties and dimensions as the above numerical model except that no burnable absorber rods are used for any assembly. A uniform source is applied on the models with fission production turned off, and the fission neutron production is tallied in each pin.

The pin-wise ratios are calculated in the bottom left assembly for each of the cases (b)-(d), as follows:

$$R_b(x,y) = \frac{f_b(x,y)}{f_a(x,y)} \quad (2)$$

Where $f_b(x,y)$ is the fission rate in pin (x,y) for the model (b). The ratios for case (a) would all be equal to one.

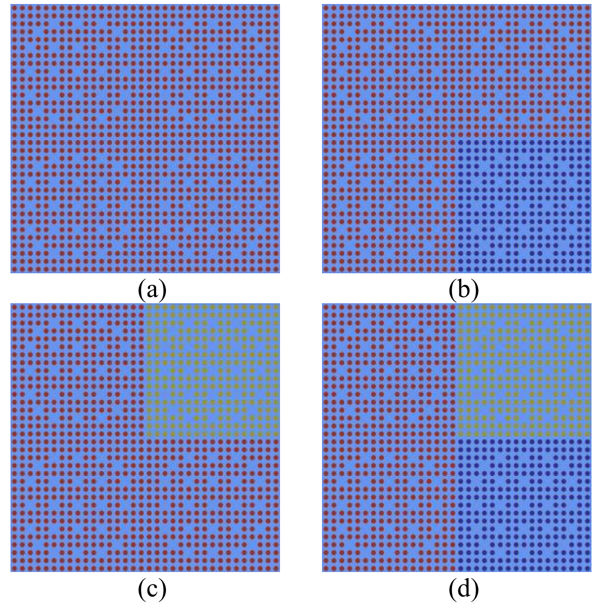
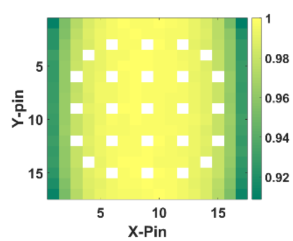


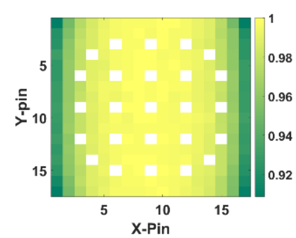
Fig. 2. The models to prove ratio multiplication technique.

In order to estimate the ratios in (d), it is proposed to multiply the ratios from (b) and (c) in order to get the combined effect of the two (i.e. $R_d(x,y) = R_b(x,y) \times R_c(x,y)$).

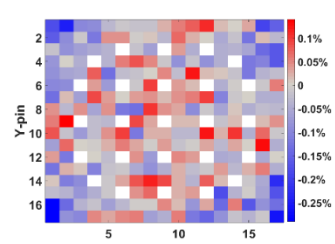
The real ratios and the multiplied ratios in the bottom left assembly and the relative errors between them are shown in Fig. 3. The highest relative error is 0.29%. It proves that the real ratios can be estimated by multiplying the ratios independently calculated from each surrounding assembly. Moreover, the burnable absorber rods are assumed to have no effects on the ratios.



(a) The real ratios.



(b) The multiplied ratios.

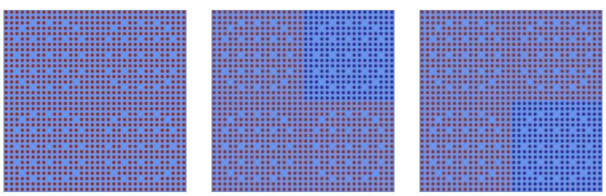


(c) The relative errors.

Fig. 3. The relative errors between multiplication ratios and real ratios.

RESULTS

The fission matrix combination method and the ratio correction fission matrix method are applied on the four-assembly model shown in Fig. 1. The fission matrices are calculated with the "fmtx" option for each of the three assembly types (using a separate model where all four assemblies in the 2×2 model are the same) and the periodic boundary are used for all the simulations. The Serpent 2 criticality neutron source distribution is taken as a reference. For the reference, 4000000 particles are used per cycle with 200 active cycles and 100 inactive cycles. For the ratio correction method, the ratios are interpolated from the ratio curves. In this work, in order to derive the ratio curves from the adjacent and the diagonal assembly, the bottom right and the top right assembly are of 3.1 $wt\%\ ^{235}U/U_{total}$ (blue) enrichment separately with all the other assemblies 1.6 $wt\%\ ^{235}U/U_{total}$ (red) enriched as shown in Fig. 4. With such ratio curves, only the ratios from 2.4 $wt\%\ ^{235}U/U_{total}$ enrichment assembly are interpolated.

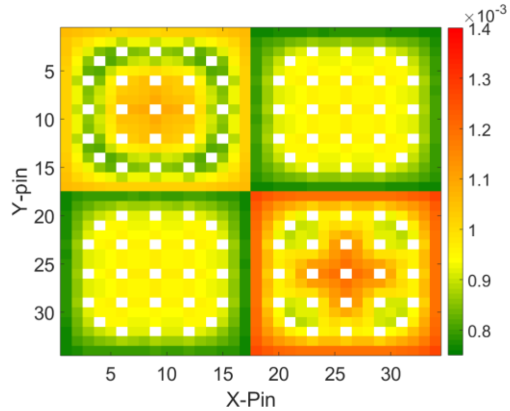


(a) (b) (c)

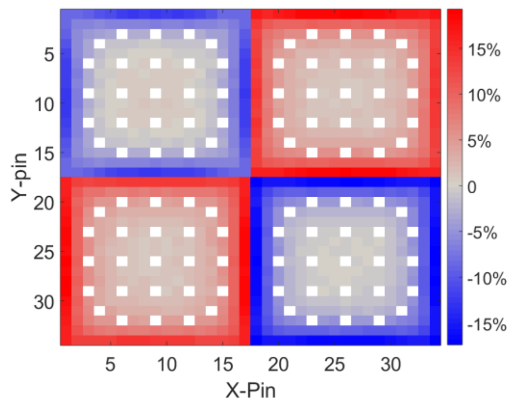
Fig. 4. The models to calculate the ratio curves.

The ratios in the bottom left assembly are calculated and the ratio curves from the top right (diagonal) assembly are derived from (b) and (a), the ratio curves from the bottom right (adjacent) assembly are derived from (c) and (a). Because of the geometric symmetry, the ratio curves from the top left assembly can also be calculated from (c) and (a). Therefore, three Monte Carlo calculations are needed to derive the ratio curves. There are six Monte Carlo calculations in total to build the ratio correction fission matrix including the other three to build the fundamental fission matrices. It has been proved that the ratio curves are almost independent of the enrichment of the assembly on which the ratios are applied [9]. Therefore, the two ratio curves are used for all the four assemblies to calculate the ratios regardless of the assembly type.

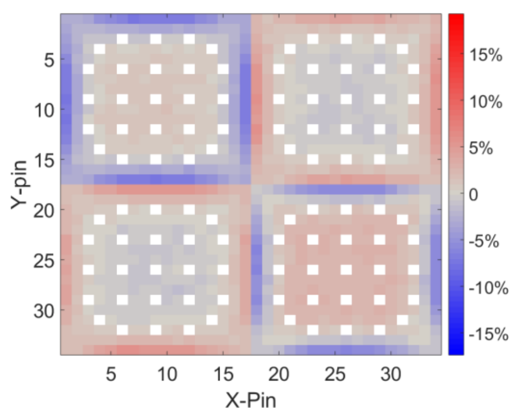
In Fig. 5, the pin-wise source distribution of the numerical model is presented as well as the pin-wise relative errors of the fission matrix methods. The root mean square of the true source tally uncertainty is 0.11%. It can be observed that the ratio correction method has greatly reduce the relative errors. The root mean square of the pin-wise relative errors for the original fission matrix combination method is 8.86%, compared to 2.52% for the ratio correction fission matrix method.



(a) True source distribution.



(b) Relative errors of the original fission matrix combination method.



(c) Relative errors of the ratio correction fission matrix method.

Fig. 5. True source distribution and the relative errors of the fission matrix methods.

In addition to the source distribution, the multiplication factors are calculated with the Serpent 2, the original fission matrix combination method (Original FM) and the ratio correction fission matrix method (Ratio FM). The multiplication factors, their difference and the Serpent calculation uncertainty are listed in Table II. The ratio correction fission matrix method also improves the accuracy of the k-eff calculation.

TABLE II. The multiplication factors with uncertainties and their difference with reference

	Serpent 2	Original FM	Ratio FM
k-eigenvalue	1.00067	0.99745	0.99920
k difference	-	-322 pcm	-147 pcm
uncertainty	4 pcm	-	-

CONCLUSIONS

The techniques to apply the ratio correction fission matrix method to the four-assembly model have been proposed. It has been shown that the ratios in an assembly m with different surrounding assemblies can be calculated by multiplying the ratios from each surrounding assembly independently. The ratios from each surrounding assembly can be linearly interpolated based on the assembly enrichment. Moreover, it is proved to be a valid assumption that the burnable absorber rods do not affect the ratios. The ratio correction fission matrix method has reported a more accurate source distribution and multiplication factor than the original fission matrix combination method. Therefore, the ratio multiplication and the assumption can be applied on the whole core ratio correction fission matrix calculations.

REFERENCES

1. KITADA, T., & TAKEDA, T. "Effective convergence of fission source distribution in Monte Carlo simulation.", *J.Nucl. Sci. Technol.*, vol. 38, no. 5, pp. 324–329, 2001.

2. W. J. WALTERS, A. HAGHIGHAT, S. SITARAMAN, and Y. HAM, "A Methodology for Determination of Detector Response for Inspection of a Spent Fuel Pool," in "*Proceedings of PHYSOR 2010, Advances in Reactor Physics to Power the Nuclear Renaissance*," Pittsburgh, Pennsylvania, May 9–14, 2010.

3. W. J. WALTERS, A. HAGHIGHAT, S. SITARAMAN, and Y. Ham, "Development of INSPCT-S for inspection of spent fuel pool," *J. ASTM Int. 1550, 690-705.*, 2012.

4. W. J. WALTERS, N. ROSKOFF, K. K. ROYSTON, and A. HAGHIGHAT, "Use of the Fission Matrix Method for Solution of the Eigenvalue Problem in a Spent Fuel Pool," in "*Proceedings of PHYSOR 2014, The Role of Reactor Physics toward a Sustainable Future*," Kyoto, Japan, September 28-October 3, 2014

5. W. J. WALTERS, "Application of the RAPID Fission Matrix Methodology to 3-D Whole-core Reactor Transport," in *M&C 2017 - International Conference on Mathematics & Computational Methods Applied to Nuclear Science & Engineering, Jeju, Korea, April 16-20*, 2017.

6. W. J. WALTERS, A. HAGHIGHAT, S. SITARAMAN, and Y. HAM, "Calculation of Sub-Critical Multiplication Using a Simplified Fission Matrix Method," *Trans. Am. Nucl. Soc.*, vol. 101, p. 447, 2009.

7. A. LAUREAU, M. AUFIERO, P. R. RUBIOLO, and D. HEUER, "Transient Fission Matrix : Kinetic calculation and kinetic parameters b eff and K eff calculation," *Ann. Nucl. Energy*, vol. 85, pp. 1035–1044, 2015.

8. A. LAUREAU, D. HEUER, P. R. RUBIOLO, M. ALLIBERT, and M. AUFIERO, "Transient coupled calculations of the Molten Salt Fast Reactor using the Transient Fission Matrix approach," *Nucl. Eng. Des.*, vol. 316, pp. 112–124, 2017.

9. D. HE and W. J. WALTERS, "A New Fission Matrix Correction Method to Estimate the Source Distribution in Nuclear Reactor Core." in " *Proceedings of PHYSOR 2018, Reactor Physics Paving the Way Towards More Efficient Systems,"* Cancun, Mexico, April 22-26, 2018

10. N. HORELIK, B. HERMAN, B. FORGET, and K. SMITH. "Benchmark for Evaluation and Validation of Reactor Simulations (BEAVRS)," v1.0.1. *Proc. Int. Conf. Mathematics and Computational Methods Applied to Nuc. Sci. & Eng.*, 2013. Sun Valley, Idaho

11. J. LEPPÄNEN, M. PUSA, T. VIITANEN, V. VALTAVIRTA, and T. KALTIAISENAHO, "The Serpent Monte Carlo code: Status, development and applications in 2013," *Ann. Nucl. Energy*, vol. 82, pp. 142–150, 2015