Application of the APEC Method to MOX Fuel Loaded PWR Cores

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INTRODUCTION

A two-step procedure based on the simplified equivalence theory (SET) [1] is the most widely used method for the PWR core analysis. In the SET method, the homogenized two-group cross-section (XS) and assembly discontinuity factor (ADF) are evaluated based on the infinite lattice and subject to noticeable, intrinsic uncertainties [2]. In our previous study, we introduced the albedo-corrected parameterized equivalence constants (APEC) method, which is a new leakage correction method applicable to the fuel assembly (FA) homogenized two-group XS of the standard two-step procedure [3]. In the APEC method, FA two-group XS is expressed as a simple polynomial function of assembly-wise current-to-flux ratio (CFR) and two-group energy spectral index (SI). Also, two-group discontinuity factors (DF) can be functionalized in terms of assembly surface-wise CFR. Reference 4 showed that two-group XSs and DFs can be well corrected during a nodal calculation and the nodal accuracy is improved a lot.

However, functionalization of DF is not as simple as functionalization of XS in the APEC method. Because DF is direction dependent while XS can be corrected by using assembly-wise integrated leakage information. In a recent study, an improved modeling of the APEC function of DF was suggested for the UOX fuel loaded small PWR core [4]. In this study, the improvement of the APEC method to correct DFs of FA in a MOX fuel loaded PWR core was studied. A 2-D method of characteristics (MOC) lattice code, DeCART2D [5] was used for reference core calculation and lattice calculations.

ALBEDO-CORRECTED PARAMETERIZED EQUIVALENCE CONSTANTS (APEC) METHOD

Functionalization of Cross-sections

The FA two-group cross-sections need to be functionalized for the APEC update during nodal calculation. Our previous study [3] showed that if a FA is symmetric, two-group homogenized cross-sections have a strong relationship with the assembly-wise CFR and two-group spectral index (SI). The assembly-wise CFR for is defined as

$$CFR_g^m = \sum_s J_{g,m}^s \Big/ \sum_s \phi_{g,m}^s \,, \tag{1}$$

where g is the group index, m is the node index, s is the surface index, J_g^s is surface net outward current and ϕ_g^s is surface average flux. And the two group SI is defined as

$$SI^m = \frac{\hat{\phi}_{F,m}}{\hat{\phi}_{T,m}} \,, \tag{2}$$

where $\hat{\phi}_{g,m}$ is node-average flux.

The cross-section changes due to the non-zero node interface condition are expressed as follows:

$$\begin{aligned}\Delta\Sigma_{x,\mathrm{F}}^m = {} & a_{1,x,\mathrm{F}} CFR_F^m + a_{2,x,\mathrm{F}} CFR_F^m \\ & + a_{3,x,\mathrm{F}} \Delta SI^m + c_{x,F} \,,\end{aligned} \tag{3a}$$

$$\begin{aligned}\Delta\Sigma_{x,\mathrm{T}}^m = {} & a_{1,\mathrm{x},\mathrm{T}} CFR_T^m + a_{2,x,\mathrm{T}} \left(CFR_T^m\right)^2 \\ & + a_{3,x,T} \Delta SI^m + c_{x,T} \,,\end{aligned} \tag{3b}$$

$$\Delta SI^m = SI^m - SI^{SA} \tag{4}$$

$$\Sigma_{x,g}^m = \Sigma_{x,g}^{SA} + \Delta\Sigma_{x,g}^m \tag{5}$$

where SI^{SA} is spectral index of the single assembly, $a_{n,x,\mathrm{g}}$ is the n^{th} coefficient for group g for reaction x. If a FA is in inner core region, $c_{x,g}$ is always zero so that the cross-section does not change if there is no neutron leakage at all. On the other hand, if a FA is neighboring baffle-reflector at outer core region, $a_{3,x,g}$ is always zero and $c_{x,g}$ has a non-zero value. The influence of the neighboring baffle-reflector on the spectrum is so large that it can be considered as a fixed constant.

Functionalization of Discontinuity Factors

In our previous study [6], the DF of a specific surface has a strong relation with the surface-wise CFR, which is defined as below:

$$CFR_g^{\mathrm{s}} = \frac{J_g^s}{\phi_g^s} . \tag{6}$$

If a FA is in inner core region, the change of thermal group DF from ADF can be expressed in the following way:

$$\Delta DF_T^s = a_{1,T,s} \times CFR_T^s, \quad (7)$$

$$\Delta DF_T^s = DF_T^s - ADF_T, \quad (8)$$

where DF_T^s is the thermal group DF at surface *s* of FA with non-zero leakages. The non-zero CFR on other sides also affect the change of DF, but is neglected here because it is relatively very small. The larger the number of parameters considered, the greater the computing cost required to determine the coefficients of the function. The fast group DF of FAs in inner core region is not functionalized for UOX fuel PWR cores.

When a FA is neighboring the baffle, the neutron leakage out of the FA is so great that the CFR on different sides should be considered in functionalization of the DF [4].

Color-Set Model Analysis

Although the APEC functions are decided for each type of XS and DF, the coefficients must be determined in order to correct XS and DF during nodal calculations. The coefficients can be obtained by fitting the data of the FA with non-zero leakage. The fitting is performed by using the least square method or by finding a unique solution.

The data of the FA with non-zero leakage are available from analyses of small color-set model. If a FA is in inner core region, a simple checkerboard type color-set model can be used. Figure 1 shows single assembly model used in the conventional assembly homogenization method and checkerboard color-set model used in the APEC method. In the checkerboard color-set model, FA 1 is surrounded by FA 2 and non-zero CFR boundary conditions are created for both FA 1 and 2. Note that the 1/8 symmetry can be used for the lattice calculation of the checkerboard color-set models.

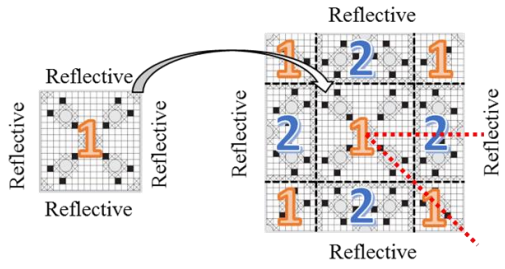

Fig. 1. Color-set model for the FA in inner core region

When a FA is neighboring baffle-reflectors, color-set model should include baffle-reflectors. Figure 2 shows two types of color-set models according to the shape of the baffle.

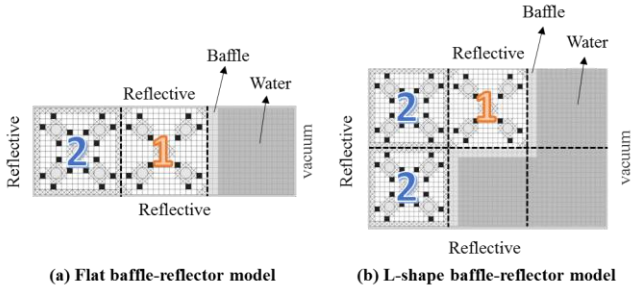

Fig. 2. Color-set models for the FA neighboring baffle

To determine the coefficients of the APEC function sufficiently accurately, we need to calculate three or more different checkerboard color-set models for each FA. For the FA neighboring baffles, 4~5 different baffle-reflector including color-set models should be calculated. The color-set models must be selected very carefully because accuracy of the fitting can be affected if the data is not well distributed over a wide range.

MOX FUEL LOADED PWR CORES

In this study, the APEC method was applied to the MOX fuel loaded KAIST-1A benchmark problem [7]. There are 3 types of UOX fuel assemblies and 2 types of MOX fuel assemblies in the core, as shown in Fig. 3. Because there are UOX FA and MOX FA together in the core, the neighborhood effect is very strong especially at interface between UOX FA and MOX FA.

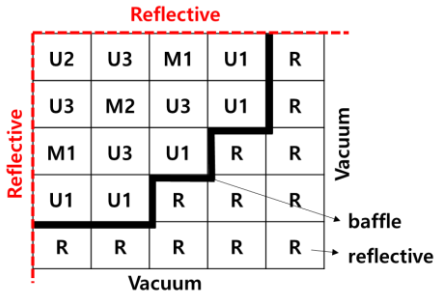

Fig. 3. KAIST-1A benchmark problem core configuration

The FA has 17x17 pins and assembly pitch is 21.42 cm. The U-235 enrichment is 2.0 w/o for U1 FA, 3.3 w/o for U2 and U3 FAs, and 0.225 w/o for M1 and M2 FAs. In M1 and M2 FA, three types of fuel pins are loaded with different Pu loading (4.3 w/o, 7.0 w/o, and 8.7 w/o). There are 16 burnable absorber (BA) pins in U2 FA and 8 BA pins in M2 FA. In U3 FA, control rods are inserted.

In addition to the original KAIST-1A benchmark problem, we considered three variant cores to obtain DF results in more environments for sensitivity analysis. The FA loading of variant cores are randomly determined by using a random number generator and their core configuration is shown in Fig. 4. However, only U1 FA was

loaded in the peripheral region to avoid undesirable FA loading near baffle-reflector, such as a) MOX FAs (M1 and M2), b) FA with a lot of BA pins (U2), and c) control rodded FA (U3).

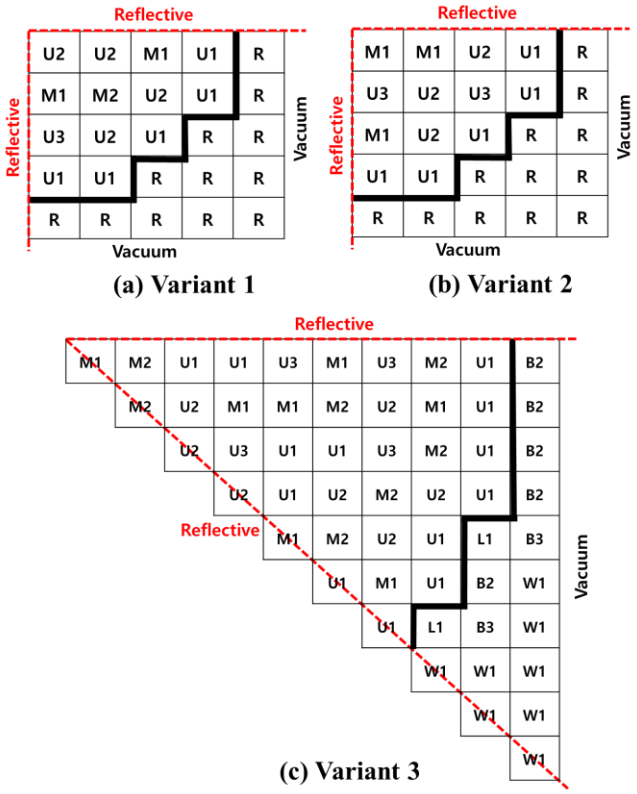

Fig. 4. Core configurations of KAIST-1A variant cores

NUMERICAL RESULT

The APEC method is implemented into an in-house nodal expansion method code in conjunction with a partial-current CMFD acceleration. The reference core calculation and lattice calculations including single assembly and color-set analysis shown in Table I are performed by using the DeCART2D code.

Cross-section Correction Result

The performance of the APEC XS correction was tested on the original KAIST-1A benchmark problem. While conventional ADF was being used, two-group XSs were updated during nodal calculations. The reference XS was obtained from the global core transport calculation by DeCART2D code. The RMS (root mean square) of relative error of FA two-group XS is compared between the conventional FWC (flux weighted constant) and the APEC corrected XS in Table II. It is clearly observed that the APEC method accurately corrected the XS even in the MOX fuel loaded core. The RMS error was reduced by more than 78%.

Table I. List of color-set calculations

Model	Set of FAs in color-set models
Checkerboard	(U2,U3), (U2,M1), (U3,M1), (U3,M2), (M1,M2)
Flat baffle	(U1,U1), (U1,U3), (U1,M1)
L-shape baffle	(U1,U1), (U1,U3), (U1,M1)

Table II. RMS error of FWC and APEC XS

	FWC (%)	APEC XS (%)
D_1	0.31	0.06
D_2	0.32	0.07
Σ_{a1}	1.40	0.11
Σ_{a2}	0.67	0.14
$\nu\Sigma_{f1}$	0.93	0.11
$\nu\Sigma_{f2}$	0.83	0.16
$\Sigma_{s1\rightarrow2}$	3.14	0.37
$\Sigma_{s2\rightarrow1}$	3.86	0.42

Functionalization of the Discontinuity Factor

Unlike the XS correction, the APEC function for DF correction could not be successfully applied to the MOX fuel loaded core. This is because the neutron energy spectrum varies greatly due to the influence of MOX fuel, so thermal group DF cannot be expressed as a simple linear function as Eq. (7). Fig. 5 shows the comparison of behavior of thermal group DF between MOX fuel loaded cores (see Fig. 3 and 4) and UOX fuel loaded SMR cores in Ref. 3.

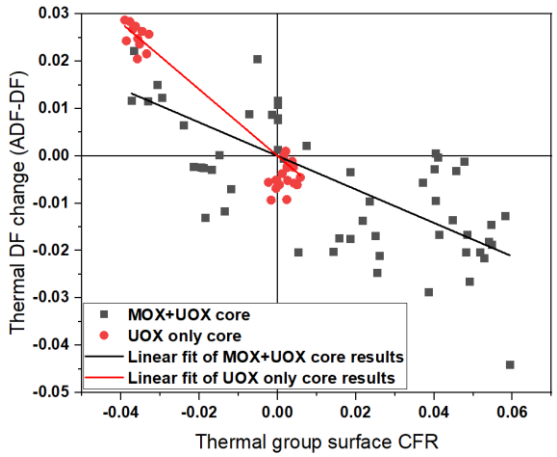

Fig. 5. Comparison of behavior of thermal group DF

It is clear that the thermal group DF of U2 FA in MOX fuel loaded cores cannot be accurately functionalized by only the CFR on the same side. Therefore, we tested various functions for the DF fitting by considering additional parameters as follows:

Fn. 1: $$\Delta DF_g^s = a_{1,g,s} \times CFR_g^s, \quad (9)$$

Fn. 2: $$\Delta DF_g^s = a_{1,g,s} \times CFR_g^s + a_{2,g,s} \times CFR_g^o, \quad (10)$$

Fn. 3: $$\Delta DF_g^s = a_{1,g,s} \times CFR_g^s + a_{2,g,s} \times \Delta SI^s, \quad (11)$$

Fn. 4: $$\Delta DF_g^s = a_{1,g,s} \times CFR_g^s + a_{2,g,s} \times CFR_g^o + a_{3,g,s} \times \Delta SI^s + a_{4,g,s} \times \Delta SI^o, \quad (12)$$

where, CFR_g^o is g^{th} group CFR on the opposite side, $SI^s = \phi_F^s / \phi_T^s$, and $SI^o = \phi_F^o / \phi_T^o$.

Eqs. (9)~(12) are fitted to DFs on 60 surfaces of 15 U2 FAs in 4 core calculations results and the fitting results are shown in Table III. RMS and Max. values of relative error of ADF and fitted DFs were obtained by using the DF value of core calculations as the reference value. Also, the performance of fitting was evaluated by calculating the GoF (goodness of fit) as Eq. (13).

$$GoF = 1 - \sum_i \left(y_i - f\left(x_i\right)\right)^2 \Big/ \sum_i \left(y_i - \bar{y}\right)^2. \quad (13)$$

Comparing the accuracy of DFs fitted by using Fn. 1~4 to that of ADF in Table III, we can see that a large accuracy improvement can be achieved by additionally considering surface SI than considering only the CFR when functionalizing DF. On the other hand, considering opposite side's surface CFR didn't improve accuracy much. Considering CFR and SI on both same and opposite sides provides higher accuracy. However, considering too many variables can increases the difficulty of the fitting and should be avoided. Also, it is noticeable that fast group DF was fitted very accurately, taking into account only two parameters by using Fn. 3. But the fitting of thermal group DF was somewhat less accurate and needs more studies.

Similarly, 4 functions are fitted to DFs on 36 surfaces of 9 MOX fuel loaded M2 FAs and the results are shown in Table IV. Again, considering SI showed significant improvement in the function fitting in terms of GoF and RMS of relative error and the fitting was more accurate for the fast group DF than the thermal group DF.

CONCLUSIONS

In this study, the APEC method which is developed for the UOX fuel loaded PWR analysis was applied to MOX fuel loaded cores. It was demonstrated that the XS correction using the APEC method in MOX cores is as effective as in the UOX cores. However, the DF functionalization in the previous APEC method was not directly applicable to the MOX core. We showed that the accuracy of the APEC DF function can be improved by considering additional parameters and the optimum APEC DF function will be determined in future work. Also, functionalization of DF using color-set analysis must be performed.

Table III. Accuracy of DF functions for U2 FA

Fast group	**ADF**	**Fn. 1**	**Fn. 2**	**Fn. 3**	**Fn. 4**
GoF	-	0.60	0.60	0.90	0.94
RMS error (%)	1.81	1.03	1.03	0.51	0.41
Max. error (%)	4.06	2.84	2.85	1.19	0.79
Thermal group	**ADF**	**Fn. 1**	**Fn. 2**	**Fn. 3**	**Fn. 4**
GoF	-	0.47	0.51	0.68	0.77
RMS error (%)	1.44	0.96	0.93	0.75	0.65
Max. error (%)	4.26	2.23	1.99	2.50	2.08

Table IV. Accuracy of DF functions for M2 FA

Fast group	**ADF**	**Fn. 1**	**Fn. 2**	**Fn. 3**	**Fn. 4**
GoF	-	0.70	0.85	0.93	0.95
RMS error (%)	1.31	0.70	0.51	0.33	0.28
Max. error (%)	3.30	1.73	1.00	0.69	0.69
Thermal group	**ADF**	**Fn. 1**	**Fn. 2**	**Fn. 3**	**Fn. 4**
GoF	-	0.07	0.17	0.46	0.67
RMS error (%)	1.22	0.80	0.75	0.61	0.48
Max. error (%)	2.63	1.80	1.90	1.95	1.67

ACKNOWLEDGMENT

This work was supported by the National Research Foundation of Korea (NRF) Grant funded by the Korean Government (MSIP) (NRF-2016R1A5A1013919)

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